
# Maximum Transfer Theorem in AC Power

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In Chapter 7 we studied this theorem when we only had resistors in a DC circuit. Now let's study it in AC and when there is presence of reactive elements in the circuit. Let's consider an impedance in series with a source of voltage, such that it is a complex impedance. This form, it will be of the type:

$$Z_i = R_i \pm j X_i \tag{1}$$

The load will also be a complex impedance. We will write as:

$$Z_L = R_L \pm X_L \tag{2}$$

To calculate the power dissipated in the load, we will first calculate the electric current that circulates through the circuit. See the circuit shown in Figure 1 for reference.

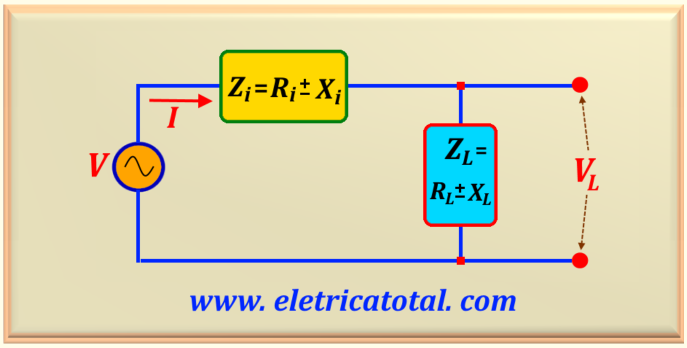


Figura 1: Circuit as reference

So, we can write that:

$$I = \frac{V}{Z_i + Z_L} \tag{3}$$

Now that we know the value of $I$, let's apply the following equation to calculate the power:

$$P = |\, Z_L \,|\; I^2 \tag{4}$$

Substituting the value of $I$ found in equation (3) into (4), we obtain:

$$P = |\, Z_L \,| \left( \frac{V}{Z_i + Z_L} \right)^2 \tag{5}$$

We can perform an algebraic manipulation on equation (5) and find:

$$P = \frac{Z_L}{(Z_i + Z_L)^2}\; V^2 \tag{6}$$

Let's assume, based on the circuit shown in Figure 1, that we have a voltage source and a fixed complex impedance in a series association, supplying power to a load. Let us consider the charge a variable complex impedance. This way, we can have three distinct cases. Let's analyze each case separately.

## Case 1 - We have $R_L \neq 0 \; e \; X_L = 0$

This is the case where the load is **purely resistive**. So let's go recalculate the value of the electric current by replacing $Z_L$ with $R_L$. Therefore:

$$I = \frac{V}{Z_i + R_L} = \frac{V}{(R_i + R_L) + j\,X_i} \tag{7}$$

But, to calculate the power, we need to know the absolute value of electrical current. So, we have:

$$|\,I\,| = \frac{V}{\sqrt{(R_i + R_L)^2 + X_i^2}} \tag{8}$$

Then the power delivered to the load $R_L$ will be:

$$P = |\,Z_L\,|\; I^2 = \frac{R_L}{(R_i + R_L)^2 + X_i^2}\; V^2 \tag{9}$$

As we want to find for which value of $R_L$ we will obtain the maximum power transfer to the load, we must calculate the first derivative of the above equation in relation to $R_L$ and equate its result to **zero**. Like this:

$$\frac{dP}{dR_L} = V^2 \left\{ \frac{[\,(R_i + R_L)^2 + X_i^2\,] - 2\,R_L\,(R_i + R_L)}{[\,(R_i + R_L)^2 + X_i^2\,]^2} \right\} = 0 \tag{10}$$

Obviously, for this expression to be null, we must have the numerator equal to **zero**. Working the numerator algebraically we find the following expression:

$$R_i^2 + 2\,R_i\,R_L + R_L^2 + X_i^2 - 2\,R_i\,R_L - 2\,R_L^2 = 0 \tag{11}$$

Performing the necessary algebraic operations to simplify the expression we arrive at:

$$R_i^2 + X_i^2 = R_L^2 \tag{12}$$

This way, we arrive at the final form, that is:

$$\boxed{R_L = \sqrt{R_i^2 + X_i^2} = |Z_i|} \tag{13}$$

Therefore, we conclude that to achieve maximum transfer of power for a purely resistive load, its value must be equal to the absolute value of the impedance of the circuit that connects the load to the voltage source.

## Case 2 - We have $R_L \neq 0$ $e$ $X_L \neq 0$

This is the case where the load has a resistive element **fixed** and reactive **variable**. Then, calculating the value of the electric current we have:

$$I = \frac{V}{Z_i + Z_L} \quad \text{ou} \quad |I| = \frac{V}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} \tag{14}$$

And the power value is given by:

$$P = R_L \, |I|^2 \tag{15}$$

Substituting equation (14) into (15), we obtain:

$$P = \frac{R_L}{(R_i + R_L)^2 + (X_i + X_L)^2} \, V^2 \tag{16}$$

Note that in this case, the value of $R_L$ is fixed. So, let's derive equation (16) with respect to $X_L$ and we will consider $R_L$ constant. Then:

$$\frac{dP}{dX_L} = V^2 \left\{ \frac{-2 R_L (X_i + X_L)}{[(R_i + R_L)^2 + (X_i + X_L)^2]^2} \right\} = 0 \tag{17}$$

As $R_L$ and $V$ have fixed values, the only way we can cancel this derivative is if:

$$\boxed{X_L = -X_i} \tag{18}$$

In other words, the load value must be the **complex conjugate** of the impedance $Z_i$.

## Case 3 - We have $R_L \neq 0$ e $X_L \neq 0$

This is the case where the load has a resistive element **variable** and reactive **fixed**. If we do the calculations for the electric current and power we will find the same values as in case 2. Therefore:

$$P = \frac{R_L}{(R_i + R_L)^2 + (X_i + X_L)^2} V^2 \tag{19}$$

Note that in this case, the value of $X_L$ is fixed. So, let's derive equation (19) with respect to $R_L$ and we will consider $X_L$ constant. Then:

$$\frac{dP}{dX_L} = V^2 \left\{ \frac{(R_i + R_L)^2 + (X_i + X_L)^2 - 2\,R_L\,(R_i + R_L)}{[\,(R_i + R_L)^2 + (\,X_i + X_L\,)^2\,]^2} \right\} = 0$$

Performing algebraic simplification operations on the numerator, we arrive at:

$$R_L^2 = R_i^2 + (X_i + X_L)^2 \tag{20}$$

Now, extracting the square root of $R_L$ we find the condition what we are looking for, that is:

$$\boxed{R_L = \sqrt{(R_i)^2 + (X_i + X_L)^2} = |\,Z_i + j\,X_L\,|} \tag{21}$$

In this way, we conclude that the maximum transfer of power to the load, when $R_L$ is equal to the absolute value of the circuit impedance.