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Single-Phase Induction Motor Impedance Calculation

by www.eletricatotal.com

Starting from the equivalent circuit of a single-phase induction motor, as shown in Figure 1, we can calculate the parallel of the resistor and reactances that appear in the circuit.

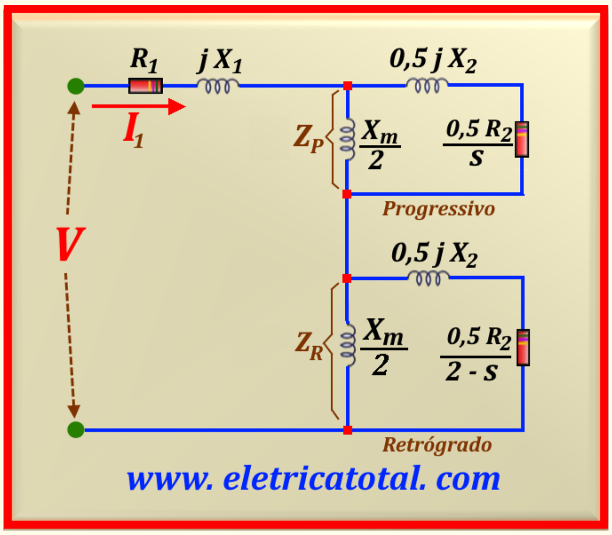

Figura 1: Reference circuit

Progressive Circuit

From the circuit above, we can easily verify that the impedance progressive, which we will call Z_P, is given by:

$$Z_P = \frac{j\,\frac{X_m}{2}\left(j\frac{X_2}{2} + \frac{R_2}{2\,s}\right)}{\frac{R_2}{2\,s} + j\,\frac{(X_m + X_2)}{2}} \tag{1.1}$$

Performing the indicated operations and, after some algebraic work, we arrive at:

$$Z_P = \frac{1}{2}\left[\frac{-\,s\,X_m\,X_2 + j\,R_2\,X_m}{R_2 + j\,s\,(X_m + X_2)}\right] \tag{1.2}$$

To eliminate the complex number in the denominator of the equation, we must multiply and divide by its complex conjugate. Thus, working algebraically, we find:

$$Z_P = \frac{1}{2}\left[\frac{s\,R_2\,X_m^2 + j\,[s^2\,X_m\,X_2\,(X_m + X_2) + R_2^2\,X_m]}{R_2^2 + s^2\,(X_m + X_2)^2}\right] \tag{1.3}$$

On the other hand, by the definition of progressive impedance, we can write it as a real part, representing progressive resistance and an imaginary part, representing progressive reactance. Thus, we have:

$$\boxed{R_P = \frac{1}{2}\left[\frac{s\,R_2\,X_m^2}{R_2^2 + s^2\,(X_m + X_2)^2}\right]} \tag{1.4}$$

$$\boxed{X_P = \frac{1}{2}\left[\frac{s^2\,X_m\,X_2\,(X_m + X_2) + R_2^2\,X_m}{R_2^2 + s^2\,(X_m + X_2)^2}\right]} \tag{1.5}$$

These equations represent the correct values of R_P and X_P. But, of course, many literatures choose to make some considerations in order to simplify these equations. Let's see what can be done. In general, we have $X_m >> X_2$. With this in mind, it is possible to write $X_m + X_2 \approx X_m$. Therefore, it is possible to write the following approximations:

$$R_P = \frac{1}{2}\left[\frac{s\,R_2\,X_m^2}{R_2^2 + s^2\,X_m^2}\right] = \frac{R_2}{2\,s\left[\left(\frac{R_2}{s\,X_m}\right)^2 + 1\right]} \tag{1.6}$$

$$X_P = \frac{1}{2}\left[\frac{s^2\,X_m^2\,X_2 + R_2^2\,X_m}{R_2^2 + s^2\,X_m^2}\right] \tag{1.7}$$

We can verify the validity of the approximations using Example 9-1, page 595, of the book Fundamentals of Electrical Machines - Chapman - 5th edition as a reference. However, it is worth noting that in the book the final value of Z_P has a printing error. Where it reads

$$Z_P = 25.4 + j\,30.7 = 39.90\angle 50.5^\circ$$

read

$$\boxed{Z_P = 28.31 + j\,31.17 = 42.11\angle 47.75^\circ}$$

The data provided in the problem are:

$$R_1 = 1.52\,\Omega \quad R_2 = 3.13\,\Omega \quad X_1 = 2.10\,\Omega$$

$$X_2 = 1.56\,\Omega \quad X_m = 58.2\,\Omega$$

Then, using our approximated equations (1.6) and (1.7), substituting in their respective numerical values, we find:

$$R_P = 14.51\,\Omega \qquad X_P = 15.97\,\Omega$$

To compare the values, we must multiply them by two, since the methodology used in the book is different from the methodology we use on the website. However, the final results are the same, because in the book, when calculating powers, the values of R_P and X_P are divided by two.
So, let's divide the values found in the book by two, obtaining:

$$R_P = 14.16\,\Omega \qquad X_P = 15.59\,\Omega$$

Note that the differences are in the decimal places, proving that it is perfectly possible to use the equations with approximation.

Retrograde Circuit

For the retrograde circuit and using Figure 1 as reference, the value for Z_R is:

$$Z_R = \frac{j\,\frac{X_m}{2}\left(j\frac{X_2}{2} + \frac{R_2}{2\,(2-s)}\right)}{\frac{R_2}{2\,(2-s)} + j\,\frac{(X_m + X_2)}{2}} \tag{1.8}$$

Performing the operations in equation (1.8) and after carrying out some algebraic work, we obtain:

$$Z_R = \frac{1}{2}\left[\frac{-X_m\,X_2\,(2-s) + j\,R_2\,X_m}{R_2 + j\,(X_m + X_2)\,(2-s)}\right]$$

By multiplying and dividing the fraction by its complex conjugate, we eliminate the imaginary term in the denominator. Thus, we obtain:

$$Z_R = \frac{1}{2}\left[\frac{(2-s)\,R_2\,X_m^2 + j\,[(2-s)^2\,X_m\,X_2\,(X_m + X_2) + R_2^2\,X_m]}{R_2^2 + (2-s)^2\,(X_m + X_2)^2}\right]$$

And by the definition of retrograde impedance, we can write it as a real part, representing retrograde resistance and an imaginary part, representing retrograde reactance. So, we get:

$$\boxed{R_R = \frac{1}{2}\left[\frac{(2-s)\,R_2\,X_m^2}{R_2^2 + (2-s)^2\,(X_m + X_2)^2}\right]} \tag{1.9}$$

$$\boxed{X_R = \frac{1}{2}\left[\frac{[(2-s)^2\,X_m\,X_2\,(X_m + X_2) + R_2^2\,X_m]}{R_2^2 + (2-s)^2\,(X_m + X_2)^2}\right]} \tag{1.10}$$

These equations represent the correct values of R_R and X_R. However, simplifying equations is a common practice in electrical engineering, especially when dealing with values that are significantly larger or smaller compared to other terms in the equation. In the case of the relationship between X_m and X_2 , if $X_m >> X_2$, it is reasonable to simplify the expression $X_m + X_2 \approx X_m$, since X_2 has a negligible impact on the final result. This approach not only simplifies the calculations but also helps focus on the components that truly affect the system's behavior. However, it is important to remember that this simplification should only be done when the difference between the values is small enough to justify the approximation without compromising the accuracy required for the analysis or application at hand. So we have the following approximations:

$$\boxed{R_R = \frac{1}{2}\left[\frac{(2-s)\,R_2\,X_m^2}{R_2^2 + (2-s)^2\,X_m^2}\right]} \tag{1.11}$$

$$\boxed{X_R = \frac{1}{2}\left[\frac{[(2-s)^2\,X_m^2\,X_2 + R_2^2\,X_m]}{R_2^2 + (2-s)^2\,X_m^2}\right]} \tag{1.12}$$

Let's use these equations and compare them with the results obtained in Chapman's book. The values from the book are:

$$Z_R = 1.51 + j\,1.56 = 2.18\angle 45.9^\circ$$

Then, using the approximated equations (1.11) and (1.12), substituting their respective numerical values, we find:

$$\boxed{Z_R = 0.8 + j\,0.8 = 1.13\angle 45^\circ\,\Omega}$$

Therefore, it is perfectly possible to use equations (1.11) and (1.12) to calculate the values of R_R and X_R, since by dividing the book values by 2 (for the reasons explained above), the errors are negligible.

Thus, the question we must answer is:

What is the advantage of using the equations developed on the website?

Using real numbers in equations, rather than complex numbers, can offer several advantages, especially in educational settings or in situations where access to scientific calculators is limited. Working with real numbers can simplify the learning process, making concepts more accessible to students beginning to explore advanced mathematics. Furthermore, operations with real numbers are more intuitive and can be performed on basic calculators, which is useful in environments where advanced technological resources are not available. However, while simplifying to real numbers can be convenient, it does not replace the need to understand and use complex numbers in appropriate contexts.