Resolution Maximum Power

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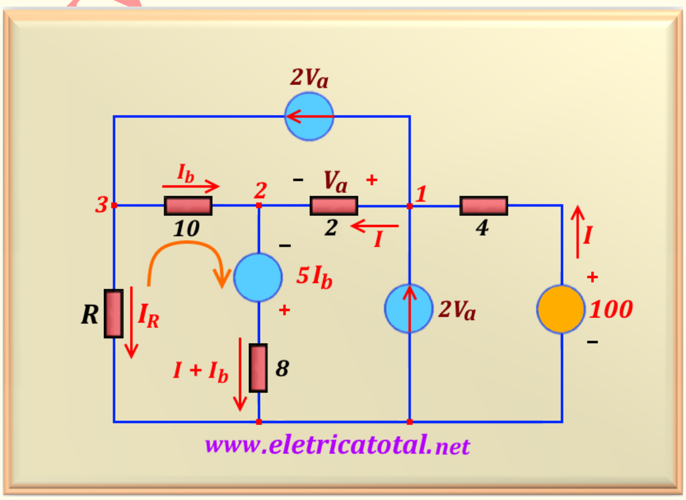
1 Enunciation

Calculate the value of R so that it dissipates the maximum power in the circuit shown below.



2 Solution

Of *problem 15-12* we have already deduced that the current I passing through the 4 ohm resistance is equal to the current passing through the 2 ohm resistance. Thus, we conclude that $V_a = 2\,I$. Then the current source of $2\,V_a$ that reaches node 3 can be replaced by the value $4\,I$. And with that, we can write that:

$$I_R \;=\; 4I - I_b \tag{1}$$

Substituting I_R for the above value we can write the mesh equation in the direction of the orange arrow shown in figure above. Thus:

$$-4\,R\,I + R\,I_b + 10\,I_b - 5\,I_b + 8\,I + 8\,I_b \;=\; 0$$

Grouping similar terms we have:

$$(8 - 4\,R)\,I + (R + 13)\,I_b = 0$$

And from this equation we find the relation between I and I_b, or:

$$I_b \;=\; \left(\frac{4\,R - 8}{R\,+\,13}\right)\,I \tag{2}$$

Substituting (2) in (1) we will find the value of I_R as a function of I. Like this:

$$I_R \;=\; 4\,I - I_b \;=\; \left(4\;-\;\frac{4\,R - 8}{R\,+\,13}\right)\,I$$

Solving this equation, we arrive at the relation below:

$$I_R \;=\; \left(\frac{60}{R\,+\,13}\right)\,I \tag{3}$$

Applying the power equation, we find:

$$P_R \;=\; R\;I_R^2 \;=\; \left(\frac{3600\;R}{(R\,+\,13)^2}\right)\,I^2$$

Algebraically rearranging the equation, we have:

$$\frac{P_R}{I^2} = \frac{3600\ R}{(R\ +\ 13)^2} \tag{4}$$

This equation shows the relation of R to the power dissipated by it and the current supplied by the independent voltage source. To determine the maximum power that will be dissipated in R requires some basic knowledge of *Advanced Calculus*.

One of the *Calculus* theorems says that:

> "*To determine the maximum or minimum of a function we must calculate the first derivative of the function with respect to the variable of interest and then equal the result to zero. And so, the maximum or minimum point of the function is determined. And to know if it is maximum or minimum, the second derivative is calculated and equals zero. If it is a negative value, the point is MAXIMUM, and if it is positive, the point is MINIMUM.*"

So let's calculate the first derivative of equation (4) with respect to the variable R. For this we must use the quotient rule given by the equation below:

$$\left(\frac{u}{v}\right)' = \frac{v\,u'\ -\ u\,v'}{v^2} \tag{5}$$

Thus, applying (5) to (4) we obtain:

$$\frac{d\left(\frac{P_R}{I^2}\right)}{d\,R} = \frac{(R+13)^2\,3600\ -\ 3600\,R\ \ 2\,(R+13)}{(R+13)^4}$$

After an algebraic work and equating the result to zero, we can write:

$$\frac{d\left(\frac{P_R}{I^2}\right)}{d\,R} = \frac{3600\,(13-R)}{(R+13)^3} = 0 \tag{6}$$

So we conclude that in order to have the maximum power dissipated in R, its value must be equal to:

$$\boxed{R = 13 \text{ ohms}}$$

To see if this value is actually maximum, we must calculate the second derivative of equation (4), equal zero, and see if its value is positive or negative. Applying (5) to (6) and after some algebraic work, we find that the second derivative equals $-0,2048$, that is, a negative value, attesting that the value found is a value of *MAXIMUM* .

To conclude, we present in the figure below the graph of equation (4) showing that the maximum of the function occurs when we have $R = 13$ ohms.