Torque x Speed in DC Series Motor

by www.eletricatotal.com

The analysis to determine the terminal characteristics of a DC series motor will be based on the assumption that the magnetization curve is linear and defined by the equation (1.1).

$$\Phi = c\, I_a \tag{1.1}$$

As a linear equation, **"c"** represents the slope of the line or constant of proportionality. This equation will be used to obtain the **torque x speed** characteristic curve of the DC motor in series configuration. We know that the induced torque of the machine is given by the equation (1.2)

$$\tau_{ind} = K\, \Phi\, I_A \tag{1.2}$$

Combining equations (1.1) and (1.2) we can write:

$$\tau_{ind} = K\, c\, I_A^2 \tag{1.3}$$

This equation tells us that the motor torque is directly proportional to the square of its armature current.

On the other hand, using Kirchhoff's voltage law (LKT) we can write:

$$V_T = E_A + I_A\,(R_A + R_S) \tag{1.4}$$

Algebraically working equation (1.3), we have:

$$I_A = \sqrt{\frac{\tau_{ind}}{K\;c}} \tag{1.5}$$

Furthermore, we know that:

$$E_A = K\;\Phi\;\omega_{\mathbf{m}} \tag{1.6}$$

Therefore, substituting equations (1.5) and (1.6) in equation (1.4), we obtain:

$$V_T = K\;\Phi\;\omega_{\mathbf{m}} + \sqrt{\frac{\tau_{ind}}{K\;c}}\,(R_A + R_S) \tag{1.7}$$

We can eliminate the flux in this equation and relate the torque of a motor to its speed. Therefore, we can rewrite equation (1.1) as follows:

$$I_A = \frac{\Phi}{c} \tag{1.8}$$

Substituting equation (1.8) in equation (1.2), we get:

$$\tau_{ind} = \frac{K}{c}\,\Phi^2 \tag{1.9}$$

Algebraically working this equation, we have:

$$\Phi = \sqrt{\frac{c}{K}}\,\sqrt{\tau_{ind}} \tag{1.10}$$

Substituting equation (1.10) in equation (1.7), we get:

$$V_T = K\,\sqrt{\frac{c}{K}}\,\sqrt{\tau_{ind}}\;\omega_{\mathbf{m}} + \sqrt{\frac{\tau_{ind}}{K\;c}}\,(R_A + R_S) \tag{1.11}$$

Performing some operations and rearranging the terms we can write:

$$\sqrt{K\ c}\,\sqrt{\tau_{ind}}\,\omega_{\mathbf{m}} = V_T - \frac{R_A + R_S}{\sqrt{K\ c}}\,\sqrt{\tau_{ind}} \tag{1.12}$$

Now we can isolate the motor speed, or:

$$\boxed{\omega_{\mathbf{m}} = \frac{V_T}{\sqrt{K\ c}}\ \frac{1}{\sqrt{\tau_{ind}}} - \frac{R_A + R_C}{K\ c}}$$

Thus, it is concluded that for an unsaturated series DC motor, the motor speed varies with the inverse of the square root of the torque.